## NONISOTHERMAL ELASTO-VISCO-PLASTIC RESPONSE OF SHELL-TYPE STRUCTURES\*

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The prediction of inelastic behavior of metallic materials at elevated temperatures has increased in importance in recent years The operating conditions within the hot section of a rocket motor or a modern gas turbine engine present an extremely harsh thermomechanical environment. Large thermal transients are induced each time the engine is started or shut down. Additional thermal transient from an elevated ambient, occur whenever the engine power level is adjusted to meet flight requirements. The structural elements employed to construct such hot sections, as well as any engine components located therein, must be capable of withstanding such extreme conditions. Failure of a component would, due to the critical nature of the hot section, lead to an immediate and catastrophic loss in power and thus cannot be tolerated. Consequently, assuring satisfactory long term performance for such components is a major concern for the designer.

Traditionally, this requirement for long term durability has been a more significant concern for gas turbine engines than for rocket motors. However, with the advent of reusable space vehicles, such as the Space Shuttle, the requirement to accurately predict future performance, following repeated elevated temperature operation, must now be extended to include the more extreme rocket motor application.

A mathematical model and solution methodologies for analyzing structural response of thin, metallic shell-type structures under large transient, cyclic or static thermomechanical loads have been developed. Among the system responses, which are associated with these load conditions, are thermal buckling and creep buckling. Thus, geometric as well as material-type nonlinearities (of high order) can be anticipated and have been considered in the development of the mathematical model. Furthermore, this was accommodated in the solution procedures.

A complete true ab-inito rate theory of kinematics and kinetics for continuum and curved thin structures, without any restriction on the magnitude of the strains or the deformation, was formulated. The time dependence and large strain behavior are incorporated through the introduction of the time rates of the metric and curvature in two coordinate systems; a fixed (spatial) one and a convected (material) coordinate system. The relations between the time derivative and the covariant derivatives (gra-

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dients) have been developed for curved space and motion, so that the velocity components supply the connection between the equations of motion and the time rate of change of the metric and curvature tenors.

The metric tensor (time rate of change) in the convected material coordinate system is linearly decomposed into elastic and plastic parts. In this formulation, a yield function is assumed, which is dependent on the rate of change of stress, metric, temperature, and a set of internal variables. Moreover, a hypoelastic law was chosen to describe the thermoelastic part of the deformation.

A time and temperature dependent viscoplastic model was formulated in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables, as well as their evolution, was motivated by phenomenological thermodynamic considerations. The nonisothermal elastic-viscoplastic deformation process was described completely by "thermodynamic state" equations.

The procedure employed permits the rates of the field formulation to be interpreted as increments in the numerical solution. This is particularly convenient for the construction of a finite element models together with incremental boundary condition histories. Finite element formulation was developed for curved beams and shells. The element matrixes were derived directly from the incrementally formulated equations using tensor oriented procedure.

Finite element solution of any boundary-value problem involves the solution of the equilibrium equation (global) together with the constitutive equation (local). Both equations are solved simultaneously in a step by step manner. The incremental form of the global and the local equation can be achieved by taking the integration over the incremental time step  $\Delta t = t_{j+1} - t_j$ . The rectangular rule has been applied to execute the resulting time integration.

For structures with unstable deformation paths (snap-buckling phenomenon), accurate and efficient description of the motion of the structure was obtained by inclusion of the inertia forces.

<u>Applications:</u> The response of a clamped circular arch and of a cylindrical panel were studied. The shallow circular clamped arch subjected to a single central concentrated load, as shown in Fig. 1, was analyzed. The material chosen for the numerical experimentation is the carbon steel C-45 (DIN 1720) with E =  $10^7$  psi, V = 0.3 and  $C_Y = 2.7 \cdot 10^4$  psi at room temperature.

The arch response, the deflection time history and the influence of temperature on the arch response are shown in Figs. 2,3 and 4, respectively.

A thin cylindrical shell panel simply supported on all sides, made of the same material as the arch, and subjected to in-plane loads along the generators as shown in Fig. 5 was also studied.

The deflection time history of the panel is shown in Fig 6, for a value of  $N_{p,p}=20$  lbs/in. This load is well below the critical (buckling) load for the geometry which is 42.15 lbs/in.

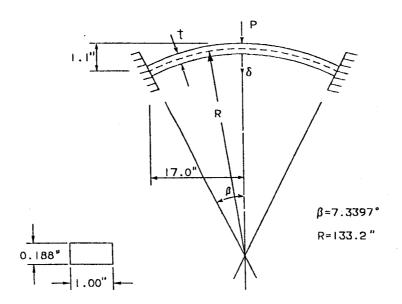


FIG. I CLAMPED CIRCULAR ARCH

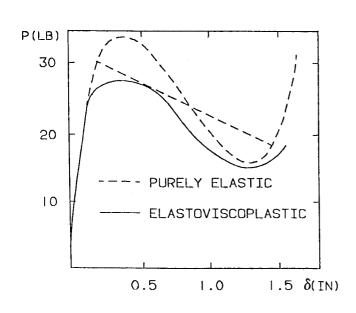


FIG. 2 THE ARCH RESPONSE

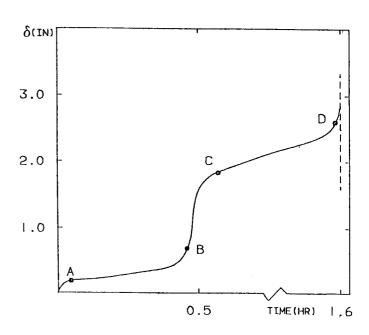


FIG. 3 DEFLECTION TIME HISTORY

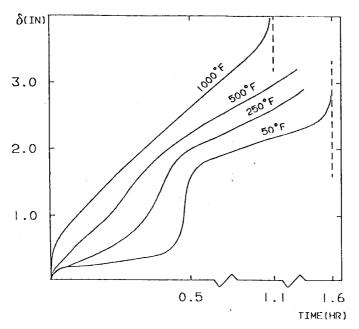


FIG. 4 THE INFLUENCE OF TEMPERATURE

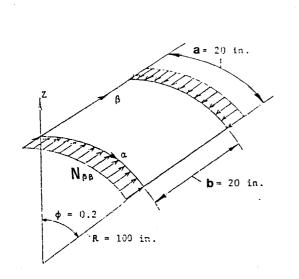
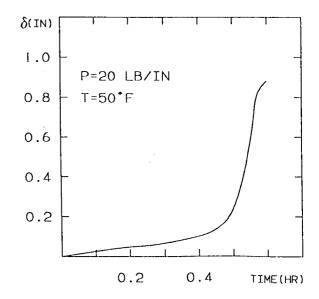


FIG. 5



CYLINDRICAL PANEL FIG. 6 CYLINDER PANEL CREEP RESPONSE